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CHARACTERISTICS AND SOME APPLICATIONS OF MODIFIED XSHANKER DISTRIBUTION

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ABSTRACT

The selection of a suitable distribution for many bio-medical real data analysis is a tedious task. The general known distributions are not a good-fit, in such cases, researchers are trying for innovative distributions to overcome such situations. Since many such real data are non-symmetric, hence they do not follow normality. In this paper, we have developed a novel version of double XShanker distribution termed as 'Modified XShanker Distribution (MOXD)' which has been described by using the weighted technique. This particular new distribution has been illustrated and explored with different statistical properties and its parameters are estimated on the basis of maximum likelihood estimation. To illustrate the predictability and flexibility of new distribution, we introduced a real lifetime biomedical data set to newly developed distribution to determine its performance over other comparable well-known distributions. Two independent real-world datasets were examined. The first dataset involved the birth weights of 130 randomly chosen new-borns from a hospital in Chennai (India). The second dataset pertained to triglyceride levels, focusing on the mean reduction (mg dL^{-1}) in triglycerides observed in 177 randomly selected patients from another Chennai hospital. These patients were monitored after taking Atorvastatin (Atorvaliq, Lipitor) continuously for 3 weeks, with triglyceride levels measured using the Cholesterol-oxidase method.

Keywords: Double XShanker distribution, maximum likelihood estimation, probability, reliability measures, weighted distribution

INTRODUCTION

There are many methods to derive new probability distributions which shows better fit than many conventional distributions. Weighted distributions are one of them and it gives the classical distribution in a new form. The idea of weighted distributions is considered to be one of the most crucial foundational aspects of distribution theory. Fisher (1934) first proposed the theory of weighted distribution to emphasize how the process of ascertainment could affect the distribution of documented observations. Furthermore, Rao (1985) illustrated this concept in a particular homogeneous situation where the standard distributions lack adequacy for attaining the observations with equal probabilities. While choosing suitable models for observed data, the idea of weighted distributions is thought to be crucial, especially when the samples are taken without the correct frame. In this case, the weighted distributions have been designed to capture the observation on the basis of a weight function. Additionally, the weighted distributions offer a collective conceptualization of the data representation and model stipulation issues. Numerous workers in their studies have made significant contributions to the development of some significant weighted probability distributions, as well as providing the illustrations for handling complex data sets from a variety of applied fields. Ganaie and Rajagopalan (2023) studied weighted power quasi-Lindley distribution and lifetime data applications. Double-weighted Rayleigh distribution was characterized and estimated by Ahmad and Ahemad (2014). Ahmad and Tripathi (2021) presented the power size-biased Maxwell distribution with the applications related to engineering data. Hassan *et al.* (2020) studied the weighted quasi-Xgamma distribution with applications to survival times. Sarma and Das (2021) proposed a weighted inverse Nakagami distribution. The length biased Devya distribution with properties along with the applications was derived by Kumar and Subramanian (2019). Hassan *et al.* (2019) introduced the weighted Ishita distribution, also with its properties and uses. The weighted Erlang distribution's characterization and information measures were proposed by Mudasir and Ahmad (2017). Rama Shanker (2015) proposed 'Shanker Distribution'. Double XShanker distribution is a recently proposed single parametric distribution which was introduced by Etaga *et al.* (2023). Amer Ibrahim and Dobbah (2023) detailed the mixture of Shanker and gamma distributions with applications to engineering data. Ray and Shanker (2023) studied about the Compound of exponential and Shanker distribution with an application.

The general known distributions are not suitable for many bio-medical data since almost all these data sets are skewed and show lack of normality. Here we derived a new asymmetrical distribution and proposed to fit a real dataset and examine the goodness. Also, we tried to calculate the structural properties like moments, characteristic function, moment generating function, etc. The maximum likelihood estimation approach was used to calculate its parameter. Also, we compared the supremacy of this new distribution with some of the known general distributions.

MATERIALS AND METHODS

Modified XShanker distribution (MOXD)

The probability density function (PDF) of double X-Shanker distribution is:

$$f(x;\theta) = \frac{\theta^2}{\left(\theta^2+1\right)^3}\left(\theta^5 + 3\theta^3 + 3\theta + x\right)e^{-\theta x};\ x > 0,\ \theta > 0 \tag{1}$$

and the cumulative distribution function (CDF) of double XShanker distribution is:

$$F(x;\theta) = 1 - \left(1 + \frac{\theta x}{\left(\theta^2+1\right)^3}\right)e^{-\theta x};\quad x > 0,\ \theta > 0 \tag{2}$$

Using a PDF $f(x)$, the random variable X represents a non-negative condition. Suppose its non-negative weight function is $w(x)$, then the PDF of weighted random variable X_w has been provided by:

$$f_w(x) = \frac{w(x)f(x)}{E(w(x))},\quad x > 0.$$

Let the non - negative weight function be $w(x)$ then $E(w(x)) = \int w(x)f(x)dx < \infty.$

For various forms of weight function *w(x)* obviously if $w(x) = x^c$, the proposed distribution is termed as weighted distribution. In this research, we have to obtain a new research of double XShanker distribution called as MOXD and its probability density function is given by:

$$f_w(x) = \frac{x^c f(x)}{E(x^c)} \tag{3}$$

Where $E(x^c) = \int_0^\infty x^c f(x)dx = \dfrac{\theta^6\Gamma(c+1) + 3\theta^4\Gamma(c+1) + 3\theta^3\Gamma(c+1) + \Gamma(c+2)}{\theta^c(\theta^2+1)^3}$ (4)

Now by utilizing the eqns (1) and (4) in eqn (3), we get the required PDF of MOXD as:

$$f_w(x) = \frac{x^c \theta^{c+2}}{\left(\theta^6\Gamma(c+1) + 3\theta^4\Gamma(c+1) + 3\theta^3\Gamma(c+1) + \Gamma(c+2)\right)}\left(\theta^5 + 3\theta^3 + 3\theta + x\right)e^{-\theta x} \tag{5}$$

and the CDF of MOXD will be determined as, $F_w(x) = \int_0^x f_w(x)dx$

$$F_w(x) = \int_0^x \frac{x^c\theta^{c+2}}{(\theta^6\Gamma(c+1)+3\theta^4\Gamma(c+1)+3\theta^3\Gamma(c+1)+\Gamma(c+2))}(\theta^5 + 3\theta^3 + 3\theta + x)e^{-\theta x}\,dx \tag{6}$$

Put $\theta x = t \quad \Rightarrow x = \frac{t}{\theta}$, Also $\theta dx = dt \ \Rightarrow dx = \frac{dt}{\theta}$, When $x \to x, t \to \theta x$ and as $x \to 0, t \to 0$

After simplification eq. 6, we would determine the CDF of MOXD as:

$$F_w(x) = \left\{\frac{1}{(\theta^6\Gamma(c+1)+3\theta^4\Gamma(c+1)+3\theta^3\Gamma(c+1)+\Gamma(c+2))}\right\}*\left\{(\theta^6\gamma(c+1,\theta x) + 3\theta^4\gamma(c+1,\theta x) + 3\theta^2\gamma(c+1,\theta x) + \gamma(c+2,\theta x))\right\} \tag{7}$$

The PDF and CDF of MOXD are represented by the following Figures (Fig. 1 and 2)..

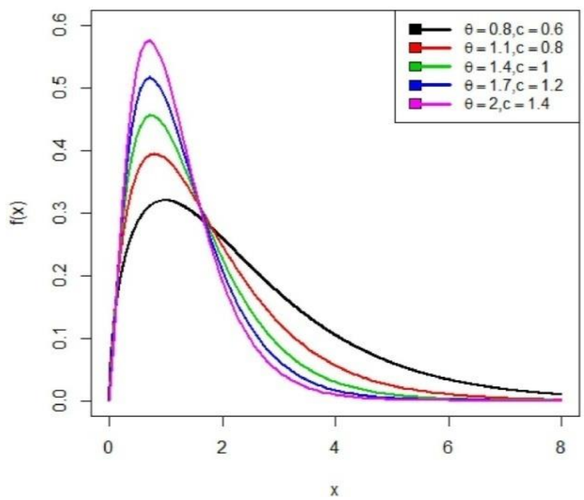


Fig. 1: Probability density function of MOXD

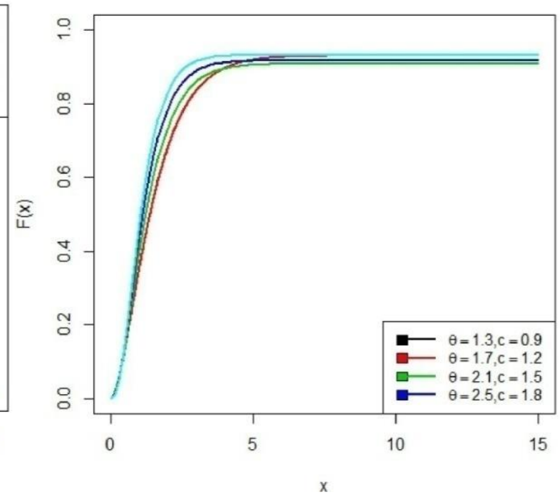


Fig. 2: Cumulative distribution function of MOXD

The nature of the PDF and CDF of MOXD is clear from the Fig. 1 and 2. Probability function shows the non-symmetric nature of the distribution. That is the distribution is positively skewed.

RESULTS AND DISCUSSION

The reliability function and the hazard rate function of the weighted double XShanker distribution will all be covered in this following given section.

Reliability function

The reliability function of MOXD will be computed as:

$$R(x) = 1 - F_w(x)$$

$$R(x) = 1 - \frac{1}{\left(\theta^6\Gamma(c+1) + 3\theta^4\Gamma(c+1) + 3\theta^3\Gamma(c+1) + \Gamma(c+2)\right)}\left(\theta^6\gamma(c+1,\theta x) + 3\theta^4\gamma(c+1,\theta x) + 3\theta^2\gamma(c+1,\theta x) + \gamma(c+2,\theta x)\right)$$

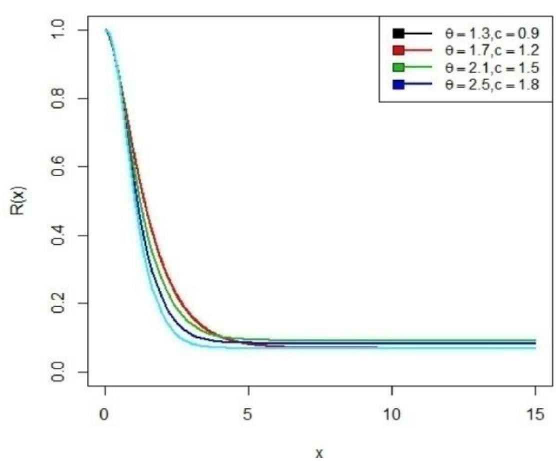
Hazard function

This is also known as failure rate or hazard rate which has been provided by:

$$h(x) = \frac{f_w(x)}{R(x)}$$

$$h(x) = \frac{x^c \theta^{c+2}(\theta^5 + 3\theta^3 + 3\theta + x)e^{-\theta x}}{(\theta^6\Gamma(c+1) + 3\theta^4\Gamma(c+1) + 3\theta^3\Gamma(c+1) + \Gamma(c+2)) - (\theta^6\gamma(c+1,\ \theta x) + 3\theta^4\gamma(c+1,\ \theta x) + 3\theta^2\gamma(c+1, \theta x) + \gamma(c+2, \theta x))}$$

The following figures show the nature of reliability (Fig. 3) and hazard rate (Fig. 4) of MOXD.



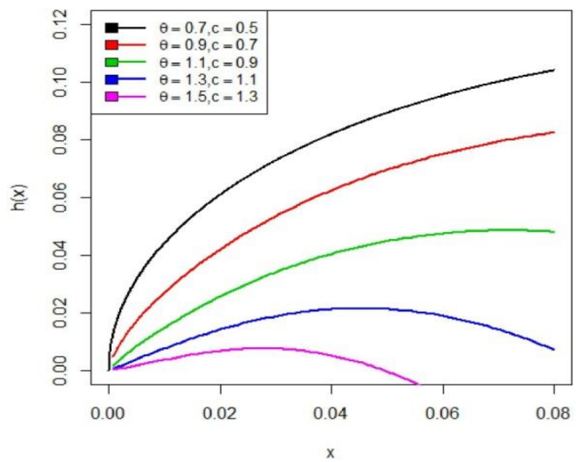


Fig. 3: Reliability of MOXD **Fig. 4: Hazard rate of MOXD**

From Fig. 4, it is clear that as ‘θ and c’ increase, the hazard rate decreases.

Structural properties: Moments and related measures

Moments

Suppose the random variable X following MOXD with parameters θ & c, then the r^{th} order moment $E(X^r)$ of introduced distribution shall be determined as:

$$E(X^r) = \mu_r' = \int_0^\infty x^r f_w(x)dx$$

$$= \int_0^\infty x^r \frac{x^c \theta^{c+2}}{\left(\theta^6\Gamma(c+1) + 3\theta^4\Gamma(c+1) + 3\theta^3\Gamma(c+1) + \Gamma(c+2)\right)}\left(\theta^5 + 3\theta^3 + 3\theta + x\right)e^{-\theta x}dx$$

$$E(X^r) = \mu_r' = \frac{\theta^{c+2}}{\left(\theta^6\Gamma(c+1) + 3\theta^4\Gamma(c+1) + 3\theta^3\Gamma(c+1) + \Gamma(c+2)\right)}\int_0^\infty x^{c+r}\left(\theta^5 + 3\theta^3 + 3\theta + x\right)e^{-\theta x}dx \tag{8}$$

After simplifying equation (8), we get

$$E(X^r) = \mu_r' = \frac{\theta^6\Gamma(c+r+1) + 3\theta^4\Gamma(c+r+1) + 3\theta^3\Gamma(c+r+1) + \Gamma(c+r+2)}{\theta^r(\theta^6\Gamma(c+1) + 3\theta^4\Gamma(c+1) + 3\theta^3\Gamma(c+1) + \Gamma(c+2))} \tag{9}$$

Now by substituting r = 1, 2, 3 and 4 in eq. 9, we would get the required ‘first four moments of” Modified XShanker distribution as:

$$E(X) = \mu_1' = \frac{\theta^6\Gamma(c+2) + 3\theta^4\Gamma(c+2) + 3\theta^3\Gamma(c+2) + \Gamma(c+3)}{\theta(\theta^6\Gamma(c+1) + 3\theta^4\Gamma(c+1) + 3\theta^3\Gamma(c+1) + \Gamma(c+2))}$$

$$E(X^2) = \mu_2' = \frac{\theta^6\Gamma(c+3) + 3\theta^4\Gamma(c+3) + 3\theta^3\Gamma(c+3) + \Gamma(c+4)}{\theta^2(\theta^6\Gamma(c+1) + 3\theta^4\Gamma(c+1) + 3\theta^3\Gamma(c+1) + \Gamma(c+2))}$$

$$E(X^3) = \mu_3' = \frac{\theta^6\Gamma(c+4) + 3\theta^4\Gamma(c+4) + 3\theta^3\Gamma(c+4) + \Gamma(c+5)}{\theta^3(\theta^6\Gamma(c+1) + 3\theta^4\Gamma(c+1) + 3\theta^3\Gamma(c+1) + \Gamma(c+2))}$$

$$E(X^4) = \mu_4' = \frac{\theta^6\Gamma(c+5) + 3\theta^4\Gamma(c+5) + 3\theta^3\Gamma(c+5) + \Gamma(c+6)}{\theta^4(\theta^6\Gamma(c+1) + 3\theta^4\Gamma(c+1) + 3\theta^3\Gamma(c+1) + \Gamma(c+2))}$$

Variance =

$$\frac{\theta^6\Gamma(c+3) + 3\theta^4\Gamma(c+3) + 3\theta^3\Gamma(c+3) + \Gamma(c+4)}{\theta^2(\theta^6\Gamma(c+1) + 3\theta^4\Gamma(c+1) + 3\theta^3\Gamma(c+1) + \Gamma(c+2))} - \left(\frac{\theta^6\Gamma(c+2) + 3\theta^4\Gamma(c+2) + 3\theta^3\Gamma(c+2) + \Gamma(c+3)}{\theta(\theta^6\Gamma(c+1) + 3\theta^4\Gamma(c+1) + 3\theta^3\Gamma(c+1) + \Gamma(c+2))}\right)^2$$

$S.D =$

$$\sqrt{\left(\frac{\theta^6\Gamma(c+3) + 3\theta^4\Gamma(c+3) + 3\theta^3\Gamma(c+3) + \Gamma(c+4)}{\theta^2(\theta^6\Gamma(c+1) + 3\theta^4\Gamma(c+1) + 3\theta^3\Gamma(c+1) + \Gamma(c+2))} - \left(\frac{\theta^6\Gamma(c+2) + 3\theta^4\Gamma(c+2) + 3\theta^3\Gamma(c+2) + \Gamma(c+3)}{\theta(\theta^6\Gamma(c+1) + 3\theta^4\Gamma(c+1) + 3\theta^3\Gamma(c+1) + \Gamma(c+2))}\right)^2\right)}$$

Harmonic mean

The "harmonic" mean of MOXD shall be determined as:

$$H.M = E\left(\frac{1}{x}\right) = \int_0^\infty \frac{x^{c-1}\theta^{c+2}}{\left(\theta^6\Gamma(c+1) + 3\theta^4\Gamma(c+1) + 3\theta^3\Gamma(c+1) + \Gamma(c+2)\right)}\left(\theta^5 + 3\theta^3 + 3\theta + x\right)e^{-\theta x}\,dx$$

$$H.M = \frac{\theta\left(\theta^5\Gamma(c+1) + 3\theta^3\Gamma(c+1) + 3\theta^2\Gamma(c+1) + \Gamma(c+1)\right)}{\left(\theta^6\Gamma(c+1) + 3\theta^4\Gamma(c+1) + 3\theta^3\Gamma(c+1) + \Gamma(c+2)\right)} \tag{10}$$

Moment generating function and characteristic function

Consider X as a random variable following MOXD with parameters θ and c, then the moment generating function of introduced distribution will be determined as:

$$M_X(t) = E\left(e^{tx}\right) = \int_0^\infty e^{tx} f_w(x)dx$$

By applying the Taylor's series, we get

$$M_X(t) = \int_0^\infty \left(1 + tx + \frac{(tx)^2}{2!} + \ldots\right) f_w(x)dx = \int_0^\infty \sum_{j=0}^\infty \frac{t^j}{j!} x^j f_w(x)dx = \sum_{j=0}^\infty \frac{t^j}{j!}\mu_j'$$

$$M_X(t) = \sum_{j=0}^\infty \frac{t^j}{j!}\left(\frac{\theta^6\Gamma(c+j+1) + 3\theta^4\Gamma(c+j+1) + 3\theta^3\Gamma(c+j+1) + \Gamma(c+j+2)}{\theta^j\left(\theta^6\Gamma(c+1) + 3\theta^4\Gamma(c+1) + 3\theta^3\Gamma(c+1) + \Gamma(c+2)\right)}\right)$$

$$M_X(t) = \frac{1}{\left(\theta^6\Gamma(c+1) + 3\theta^4\Gamma(c+1) + 3\theta^3\Gamma(c+1) + \Gamma(c+2)\right)}\sum_{j=0}^\infty \frac{t^j}{j!\,\theta^j}\left(\theta^6\Gamma(c+j+1) + 3\theta^4\Gamma(c+j+1) + 3\theta^3\Gamma(c+j+1) + \Gamma(c+j+2)\right)$$

Similarly, the MOXD's characteristic function will be determined as:

$$\varphi_X(t) = M_X(it)$$

$$M_X(it) = \frac{1}{\left(\theta^6\Gamma(c+1)+3\theta^4\Gamma(c+1)+3\theta^3\Gamma(c+1)+\Gamma(c+2)\right)}\sum_{j=0}^{\infty}\frac{it^j}{j!\,\theta^j}\left(\theta^6\Gamma(c+j+1)+3\theta^4\Gamma(c+j+1)+3\theta^3\Gamma(c+j+1)+\Gamma(c+j+2)\right)$$

Maximum likelihood estimation and Fisher's information matrix

The maximum likelihood estimation method for "estimating the parameters of MOXD" and observing its Fisher's information matrix will be covered in this section. If $X_1, X_2..., X_n$ are random samples of size n from the weighted double XShanker distribution, the likelihood function is defined as:

$$L(x) = \prod_{i=1}^{n} f_w(x)$$

$$\prod_{i=1}^{n}\left(\frac{x_i^{\,c}\,\theta^{c+2}}{\left(\theta^6\Gamma(c+1)+3\theta^4\Gamma(c+1)+3\theta^3\Gamma(c+1)+\Gamma(c+2)\right)}\left(\theta^5+3\theta^3+3\theta+x_i\right)e^{-\theta x_i}\right)$$

The log likelihood function could be expressed as:

$$\log L = n(c+2)\log\theta - n\log\left(\theta^6\Gamma(c+1)+3\theta^4\Gamma(c+1)+3\theta^3\Gamma(c+1)+\Gamma(c+2)\right)+c\sum_{i=1}^{n}\log x_i + \sum_{i=1}^{n}\log\left(\theta^5+3\theta^3+3\theta+x_i\right)-\theta\sum_{i=1}^{n}x_i \tag{11}$$

By differentiating the log likelihood eq. 11 with respect to parameters θ & c, we get the following normal equations as:

$$\frac{\partial\log L}{\partial\theta} = \frac{n(c+2)}{\theta} - n\left(\frac{6\theta^5\Gamma(c+1)+12\,\theta^3\Gamma(c+1)+9\,\theta^2\Gamma(c+1)}{\left(\theta^6\Gamma(c+1)+3\theta^4\Gamma(c+1)+3\theta^3\Gamma(c+1)+\Gamma(c+2)\right)}\right)$$

$$+\sum_{i=1}^{n}\left(\frac{5\theta^4+9\theta^2+3}{\left(\theta^5+3\theta^3+3\theta+x_i\right)}\right)-\sum_{i=1}^{n}x_i = 0$$

$$\frac{\partial\log L}{\partial c} = n\log\theta - n\psi\left(\theta^6\Gamma(c+1)+3\theta^4\Gamma(c+1)+3\theta^3\Gamma(c+1)+\Gamma(c+2)\right)+\sum_{i=1}^{n}\log x_i = 0$$

It is not possible to solve the aforementioned likelihood equations algebraically. As a result, we calculate the necessary parameters of the developed distribution by utilizing an approach similar to the Newton-Raphson method.

In order to apply the asymptotic normality outcomes for determining the confidence interval. We have that if $\hat{\alpha} = (\hat{\theta}, \hat{c})$ represents the MLE *of* $\alpha = (\theta, c)$. We can rectify the results as

$\sqrt{n}(\hat{\alpha}-\alpha) \to N_2(0, I^{-1}(\alpha))$ Where $I^{-1}(\alpha)$ is Fisher's informatio n matrix.i.e.,

$$I(\alpha) = -\frac{1}{n}\begin{pmatrix} E\left(\frac{\partial^2\log L}{\partial\theta^2}\right) & E\left(\frac{\partial^2\log L}{\partial\theta\,\partial c}\right) \\ E\left(\frac{\partial^2\log L}{\partial c\,\partial\theta}\right) & E\left(\frac{\partial^2\log L}{\partial c^2}\right) \end{pmatrix}$$

Simulation analysis

The simulated data from the PDF of MOXD is analyzed with descriptive statistics given in Table 1.

Table 1: Descriptive statistics of the simulated data with MLE of θ = 0.2190 and c = 0.9901

Mean	5.9007	Skewness	0.7339
Standard error	0.1866	Range	8.3
Median	5.5	Minimum	2.7
Mode	4.7	Maximum	11
Standard deviation	2.1604	Sum	790.7
Sample variance	4.6671	Count	5000
Kurtosis	0.6618	Confidence level (95.0%)	0.3691

Table 1 shows that the simulated data is asymmetrically distributed and such real-life situations can be analyzed by assuming that these data follow MOXD which indicates the relevance of the same.

Applications of Modified XShanker distribution

To assess the goodness of fit of two extremely different real biomedical lifetime dataset in MOXD, we examined and applied it to birth weight and then to triglycerides data.

Birth weight refers to the weight of a new-born immediately after birth. It is a critical measure used to assess the health and well-being of a baby. Birth weight is influenced by multiple factors including genetics, maternal health, prenatal care, and environmental conditions. The birth weight as categorized into three groups viz., low birth weight < 2500 g; normal birth weight: between 2500-4,000 g and high birth weight (macrosomia) > 4000 g.

Triglycerides are a type of fat found in human blood. From food, the body converts any calories, which it doesn't need to use right away into triglycerides. The triglycerides are stored in fat cells. Later, hormones release triglycerides for energy between meals. Normal range of triglycerides in human is < 150 mg dL^{-1} (< 1.7 m moles L^{-1}), borderline high level is in between 150-199 mg dL^{-1} (1.8 to 2.2 m mol L^{-1}) and high is 200-499 mg dL^{-1} (2.3 to 5.6 m mol L^{-1}) [Hinou, 2024]. A triglycerides test is usually done as a part of a group of tests called a ‘lipid profile’. It measures the level of fats in blood, including triglycerides and cholesterol.

The birth weight of 130 randomly selected new-born babies was the first set of data. The mean reduction in triglycerides (method: cholesterol-oxidase) after taking the medicine Atorvastatin (Atorvaliq, Lipitor) for 3 weeks continuously was the variable of concern here to test the goodness of fit. The distribution of birth-weight and mean reduction of triglycerides were studied and characterisation were derived with respect to many probability distributions. In both cases the statistical concepts and reality should be synchronised if they are to be accepted.

Dataset 1: The birth weight of 130 randomly selected new-born babies in Chennai (India) was noted from January 2023-June 2024 (Table 2).

Table 2: The birth weight of 130 randomly selected new-born babies at a hospital in Chennai (India)

3.480	1.560	1.910	2.625	3.580	1.940	1.910	2.630	3.595	2.610
2.195	1.910	2.630	3.595	2.610	3.480	2.555	1.910	2.625	3.580
3.115	4.450	2.160	2.225	3.120	4.485	1.965	2.230	3.165	2.225
1.870	2.400	3.450	2.300	1.900	2.500	3.500	2.300	1.900	1.700
1.910	2.650	3.700	1.250	1.980	2.660	3.780	1.250	2.010	1.700
3.925	1.800	2.040	2.825	4.035	1.280	2.085	2.890	4.120	2.690
2.090	2.900	4.170	1.940	2.090	2.935	4.240	1.800	2.135	1.930
4.255	1.505	2.155	2.965	4.280	1.950	2.190	3.000	4.305	2.960
2.195	3.100	4.370	1.615	2.220	3.150	4.450	1.960	2.225	1.750
4.485	1.965	2.230	3.165	4.570	1.965	2.300	3.345	4.605	3.115
2.325	3.375	4.665	1.940	2.095	2.935	4.240	1.850	2.135	1.770
3.480	2.555	1.910	2.625	3.580	0.940	1.910	2.630	3.595	2.610
3.480	2.550	2.190	2.625	2.570	2.195	3.105	4.375	2.065	2.610

Source: *https://figshare.com/articles/dataset/Birth_Weight_Data_sav/12315734?file=22701056*

Dataset 2: The real lifetime data, from 77 randomly selected patients in Chennai, which shows the mean reduction (mg dL^{-1}) of triglycerides (method: cholesterol-oxidase) after taking the medicine Atorvastatin (Atorvaliq, Lipitor) for 3 weeks continuously (Table 3).

Table 3: The mean reduction of triglycerides (mg dL^{-1}) from patients attending a hospital at Chennai

7.9	7.7	8.1	8.3	8.5	8.6	8.7	8.9	9.2	9.3	9.3
5.6	5.8	5.9	6.0	6.1	6.0	6.2	6.4	6.3	6.5	6.9
6.6	6.5	6.5	6.5	6.7	6.7	6.8	6.9	6.9	6.9	6.9
7.0	7.2	7.5	7.4	7.5	7.4	7.5	7.6	7.5	7.8	7.9
7.9	7.9	8.2	8.3	8.5	8.6	8.7	8.8	9.2	9.3	9.2
9.7	10.1	10.5	11.0	6.5	6.5	6.5	6.5	6.6	6.7	6.9
2.9	4.2	4.2	4.5	4.7	4.7	9.7	10.1	10.5	1.0	6.5

Source: *https://www.ncbi.nlm.nih.gov/*

The distribution of both data sets was checked with Shanker, Garima and Lindley distributions. Then it was tested with the modified double XShanker distribution. The statistical analysis is as follows. We used the criteria values like BIC (Bayesian information criterion), AIC (Akaike information criterion), AICC (Akaike information criterion corrected), and -2logL to compare the performance of MOXD over double XShanker, Shanker, Garima, and Lindley distributions. The distribution was considered better if it had lesser criterion values of *BIC*, *AIC*, *AICC* and *-2logL*. For computing the criterion values of *BIC*, *AIC*, *AICC* and *-2logL*, the following formulas were applied. Where *logL* is the maximized value of log-likelihood function, *n* is the sample size and *k* as a number of parameters in statistical model. Here AIC = 2k -2 log L, BIC = k log n – 2 log L and AICC = AIC + $\frac{2K(K+1)}{N-K-1}$

Table 3: Distribution fitting -MLE, S.E, criteria (AIC, BIC, AICC, -2logL, CI, KS, P)

Distribution	MLE	SE	-2logL	AIC	BIC	AICC	CI 95%	KS	P-value
MOXD	$\hat{\theta} = 1.6230$	$\hat{\theta} = 0.2220$	268.93	272.89	277.75	273.04	(1.54, 1.93)	0.0290	0.7977
	$\hat{c} = 1.8350$	$\hat{c} = 0.5411$							
DX	$\hat{\theta} = 0.9101$	$\hat{\theta} = 0.0531$	288.54	290.50	292.88	290.50	(0.86, 0.96)	0.0385	0.7709
Shanker	$\hat{\theta} = 1.2121$	$\hat{\theta} = 0.0541$	285.61	287.56	289.99	287.61	(1.07, 1.18)	0.1446	0.6301
Garima	$\hat{\theta} = 1.2140$	$\hat{\theta} = 0.0613$	283.71	285.69	288.12	285.75	(1.09, 1.19)	0.1450	0.6205
Lindley	$\hat{\theta} = 0.6501$	$\hat{\theta} = 0.0463$	299.37	301.37	303.80	301.42	(0.62, 0.68)	0.1115	0.6409

DX - Double XShanker, SE - Standard error, KS – Kolmogorov-Smirnov statistic, CI - confidence interval.

Table 4: MLE, SE, Criterions (AIC, BIC, AICC, -2logL) of mean reduction of triglycerides (mg dL^{-1})

D	MLE	SE	-2logL	AIC	BIC	AICC	CI 95%	KS	P value
MOXD	$\hat{\theta} = 0.225$	$\hat{\theta} = 0.0539$	178.08	182.08	184.88	182.52	(0.2137, 0.2362)	0.0291	0.7901
	$\hat{c} = 0.901$	$\hat{c} = 0.4296$							
DX	$\hat{\theta} = 0.2104$	$\hat{\theta} = 0.0254$	184.18	186.18	187.58	186.32	(0.1653, 0.2554)	0.0389	0.7791
Shanker	$\hat{\theta} = 0.2211$	$\hat{\theta} = 0.0275$	186.65	188.65	190.05	188.80	(0.1760, 0.2661)	0.0409	0.6977
Garima	$\hat{\theta} = 0.1675$	$\hat{\theta} = 0.0261$	187.55	189.55	190.95	189.70	(0.1591, 0.1758)	0.1499	0.0589
Lindley	$\hat{\theta} = 0.2026$	$\hat{\theta} = 0.0263$	183.78	185.78	187.18	185.92	(0.1924, 0.2127)	0.0912	0.0595

The Tables 3 and 4 above indicate that MOXD has lower BIC, AIC, AICC, and -2logL values as compared to the double XShanker, Shanker, Garima, and Lindley distributions. Hence, for such biomedical real datasets, it revealed that MOXD provides a quite satisfactory fit over double XShanker, Shanker, Garima and Lindley distributions.

Conclusion: In this research, a new generalization of double XShanker distribution [MOXD] has been introduced, which incorporated two parameters. The developed distribution was formed by using weighted approach to its baseline distribution. Throughout this comprehensive analysis, several structural properties of MOXD were explored and described. These characteristics included moments, mean and variance, shape of the PDF and CDF, order statistics, reliability function and hazard rate function. Additionally, the maximum likelihood estimation approach was used to estimate the proposed distribution parameters. The birth weight of randomly selected new-born babies was the first variable and the mean reduction in triglycerides [after taking medicine Atorvastatin (Atorvaliq, Lipitor) for 3 weeks continuously] was the other variable to test the goodness of fit. The distribution of the birth-weight and the mean reduction in triglycerides were studied and their characterisation were derived with respect to some probability distributions. The superiority of MOXD was examined by applying this asymmetrical real biomedical data set. It is seen from the outcomes that MOXD provides a significant fit over double XShanker, Shanker, Garima and Lindley distribution for such asymmetrical real data.

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